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ATOM OPTICS

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INTRODUCTION

By atom optics we mean the rich collection of emerging techniques by which atoms may be manipulated in the manner of light in classical optics. Existing atom optical elements include mirrors, lenses, and diffractive optics including beam splitters as well as dissipative elements such as slowers, 'coolers', and traps which have no analogue in classical optics. To date, these atom optical elements have been realized as demonstrations of principal, we hope that we will soon see some of them used as tools in real experiments. We must caution the reader that this paper is intended as a introduction and enticement to atom optics, not as an exhaustive survey. Most of the paper will be devoted to atom interferometers; first general comments on beam splitters and interferometer geometries, then a detailed look at the one we are currently constructing, and finally a discussion of a few possible experiments with atom interferometers. The final section of the paper will describe an assortment of atom optical elements, concluding with a return to nearer term experimental realities — the need for the rapid development of atom sources that are both slow *and* bright.

ATOM INTERFEROMETERS

Gratings

The key component necessary for the construction of an atom interferometer is a coherent beam splitter. Therefore we will first discuss the available atom beam splitters with special regard to their suitability for constructing an atom interferometer.

Due to the large potential energy of atoms in solids the tunnelling depth of a free atom with thermal energy is less than atomic dimensions; thus, beam splitters based on partial transmission appear impossible. We now list three general classes of beam splitters for atoms.

1) Reflective diffraction gratings. Although it was not perceived as such, the first atomic beam splitter was demonstrated in 1929[1]; it was the diffraction of atoms from the surface of ionic crystals. Because the interatomic spacing in a crystal surface is of the same order as the de Broglie wavelength (λ_{dB}) of typical atomic beams, the angular separation of the diffracted beams is of order unity (i.e. ~1 rad). Atoms may be specularly reflected by surfaces when the de Broglie wavelength corresponding to the momentum perpendicular to the surface is much lager than the surface roughness. It should be possible to use this effect to diffract atoms by grazing incidence reflection from a high quality laminar grating. We are currently trying to demonstrate this type of atom diffraction grating.

2) Transmission diffraction gratings. In 1983 our group demonstrated the Kapitza-Dirac effect in which atoms are diffracted from a standing wave of near resonant light[2]. The grating period in the standing wave is 1/2 the optical wavelength, thus the angular separation

of the diffracted orders is $2\lambda_{dB}/\lambda_{light}$ which is ~60 μrad for a thermal sodium beam. In 1988 we demonstrated the diffraction of atoms by transmission through a fabricated periodic structure[3]. The transmission gratings are arrays of slits with a spatial period of 0.2 μm in a 0.5 μm-thick gold membrane.

3) Conventional beam splitters. If one could make a transmission grating micro structure with a surface sufficiently smooth to reflect atoms incident at some grazing angle while still transmitting atoms through the slits then one would have a near analog to the half silver beam splitter used in conventional optics. Unless the grating period is sufficiently small, such a device will still waste about 1/2 of the flux by scattering atoms into orders other than the desired 0^{th} order reflected and transmitted beams.

Interferometer Geometries

Interferometers have different geometries and properties depending on what class of grating is used. Irrespective of the class of grating used, the poor velocity width of existing atom beam sources ($\Delta v/v \sim 1\text{-}10^{-3}$) force one to design a white fringe interferometer in which an achromatic central fringe is assured by using equal path lengths on either side of the interferometer. We now define various quantities needed for the discussion of interferometer properties; the atom de Broglie wavelength (λ_{dB}), the angle of incidence of the atom beam on the grating measured with respect to the grating surface (θ), and the grating period p. The height (h) (measured along the grating lines) and width (w) of the beam are also needed to determine the requirements on flatness and alignment. The various requirements on relative alignment of the gratings are of two types. The first is on the flatness of the gratings and the relative alignment of the grating surfaces, that is the collinearity of the vectors normal to the grating surfaces. The second is the alignment of the grating lines, that is the relative alignment of the gratings with respect to rotations about the surface normals.

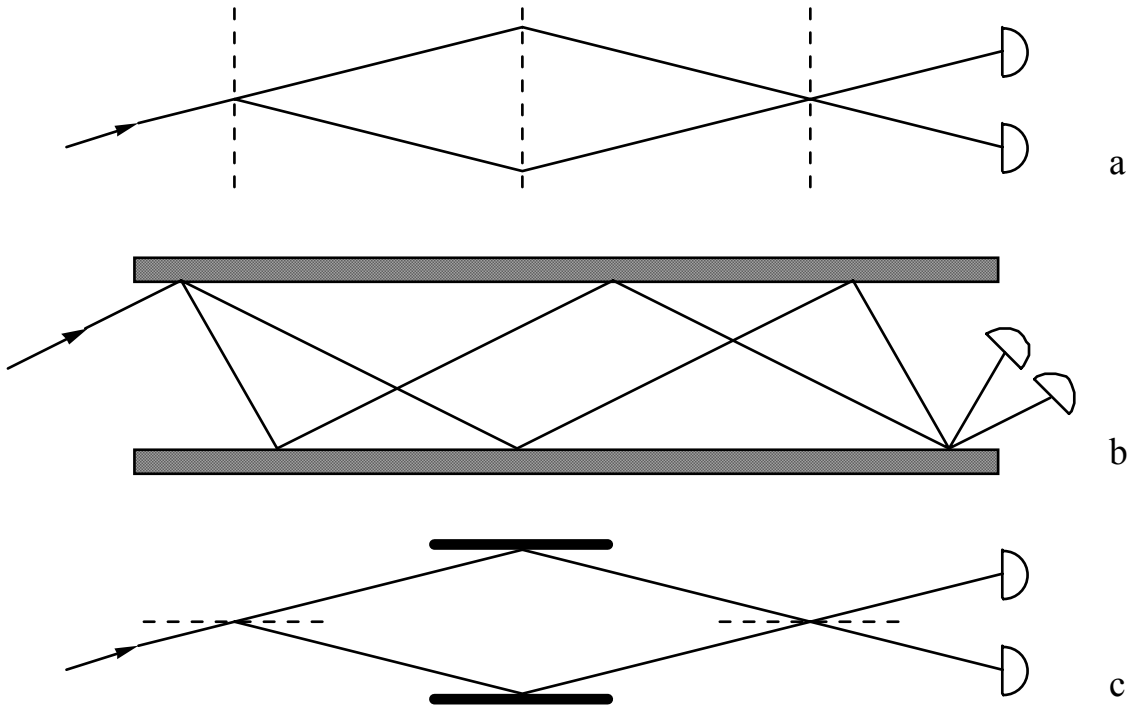

Figure 1: Three different interferometer geometries, transmission gratings (a), reflection gratings (b), and thin reflective gratings (c). In all cases angles are greatly exaggerated and diffracted beams that do not end at one of the detectors are not shown.

For transmission gratings it is well known that an arrangement of three equally spaced gratings has may desirable properties[4]. This (Fig. 1a) is the same geometry as is used for neutron interferometers. This type of interferometer is completely insensitive to the incident angle and is achromatic, all requirements on relative alignment are on the scale of the grating period — independent of , and much smaller than, λ_{dB}. The grating lines must be parallel to

$\sim p\sin(\theta)/h$, and the requirement on grating surface alignment expressed as a requirement on $\Delta\theta$ is that $\Delta\theta < p\sin^2(\theta)/w\cos(\theta)$.

For reflection gratings the dependance of the separation of the 0th and 1st order diffracted beams on the angle of the incident beam make it harder to find geometries which are white fringe[5]. Figure 1b shows an example of a white fringe geometry for reflection gratings. The requirement on surface alignment is λ_{dB}/d where d is the larger of h and w. The conditions on line alignment are the same as for transmission gratings, $\sim p\sin(\theta)/h$.

The properties of an atom interferometer made with 'conventional' beam splitters (fig. 1c) are different in several important respects. The requirement on grating surface alignment is λ_{dB}/d where d is as above, this is independent of the grating period. There is no requirement on the grating line alignment. Unlike the two previous cases the area of a conventional beam splitter interferometer is independent of λ_{dB}, which is important when one considers using atom interferometers as rotation sensors.

Our Interferometer

We are currently constructing a three transmission grating interferometer for sodium atoms. We now turn to a detailed description of this interferometer with the hope that the problems involved have some general interest. The present (late October 1989) state of this experiment is that all of its components have worked once, and we are hard at work. Our interferometer differs from the design described above only in that the the interference is detected as a spatial variation of particle density at the third grating, rather than by the variation in intensity in two beams with different directions of propagation in the far field. This detection scheme is of course only possible with amplitude gratings, it has the advantage that it requires only 2/3 the length of the separated beam method which gives us 3/2 greater separation of the beams in the interferometer for the fixed length of our beam tube.

The interferometer is built with a grating spacing of 60 cm giving us a 60 μm beam separation at the middle grating. This allows us to completely separate a 30 μm wide beam which would have an intensity of $\sim 10^6$ sec^{-1} using our existing apparatus with no gratings in place. A realistic estimate of our anticipated final signal strength may therefor be obtained from the properties of the individual gratings. Attenuation caused by the primary grating and the grating support structure gives an intensity in the 0th order of 1/8 of the incident intensity, and of 1/16 in each of the $\pm 1^{st}$ orders. These factors combine to give an intensity at the maximum of a fringe after transmission through all three gratings of only 0.005 of the incident intensity. The near field detection scheme limits the theoretical fringe contrast to 4:1, resulting in a final interference signal of ~0.004 of the incident intensity. Thus, the final interference signal through the interferometer is anticipated to be at most $\sim 4 \infty 10^3$ sec^{-1}: this signal will be reduced by any misalignment of the gratings. This signal greatly exceeds the noise of the detector ~10 sec^{-1}, allowing us in principal to see the fringes with a S/N of ~4 after a 0.01 sec averaging time.

There are a variety of experimental complications not mentioned in this description of our interferometer. We will now discuss the two of these which appear the most problematical and which are likely to be problems in any atom interferometer: vibration isolation and grating alignment.

We begin our discussion of vibration isolation with a review of the vibration problems relevant to our interferometer. There are two requirements, the first is that the three gratings are stationary relative to each other to within ~1/4 period (50 nm) during the time the final grating integrates the intensity at a given position. Thus, the rms amplitude of relative vibrations integrated over all frequencies greater than the reciprocal of the integration time must be less than ~50 nm. The second requirement is on motion of the gratings as a unit due to acceleration of the center of mass of the grating system during the time it takes for the atoms to traverse the interferometer, the motion due to this acceleration must also be less than ~1/4 period. In our interferometer the transit time is 1.3 msec which implies that the rms acceleration below ~900 Hz must be less than 10^{-2} ms^{-2}.

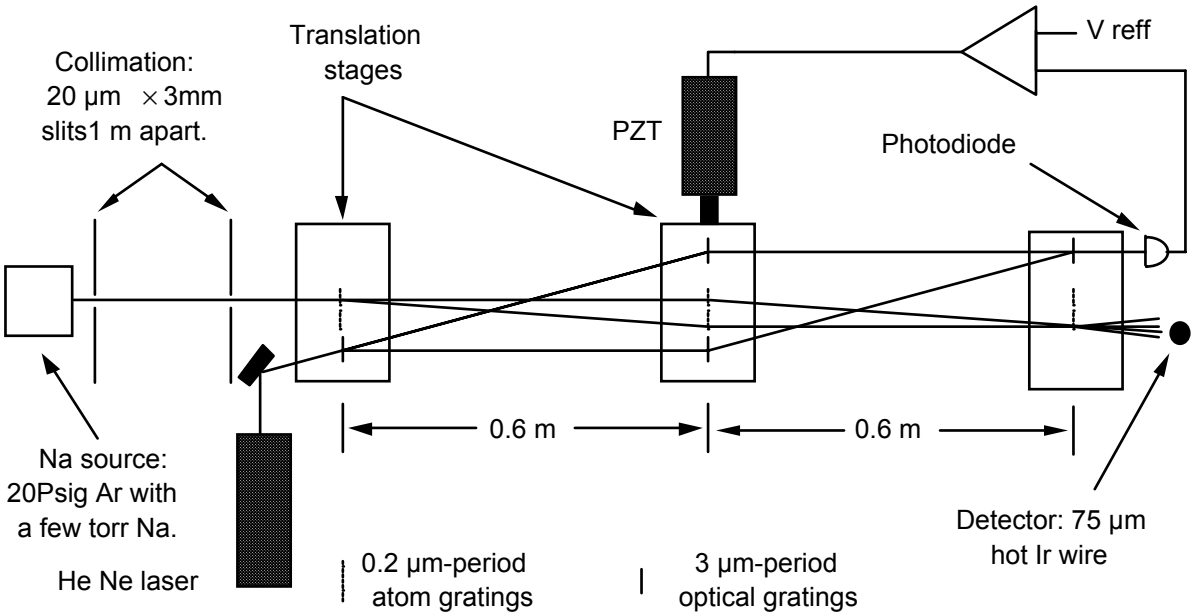

Figure 2: A schematic of our interferometer showing the active vibration isolation system. Not to scale.

We have attacked our vibration problem using a combination of passive isolation and active feedback. The passive isolation system consists of small pneumatic feet which support the apparatus and act like damped springs with a 2 Hz resonant frequency. This simple isolation system reduced the rms motion due mainly to building noise by an order of magnitude to ~ 0.5 μm. The active feedback system is used to stabilize the relative positions of the three gratings at frequencies below ~150 Hz. This system works best at low frequencies (< 10 Hz) where the passive system is least effective. The reduction of relative motion provided by the active system will allow us to use much longer integration times when we are looking for the interference signal. The active feedback system uses a laser interferometer which has the same transmission grating geometry as the atom interferometer. The gratings for the optical interferometer are mounted on the same three translation stages as the matter wave gratings in order to record the exact relative alignment of the matter wave interferometer. The error signal from the optical interferometer provides a measure of the relative alignment of the three grating platforms, it is applied to a Peazo-electric translator (PZT) through a feedback network in order to stabilize the platforms. Using this system we have reduced the relative rms motion of the gratings from ~1500 to 40 nm.

In order that all points along the height (3 mm) of our ribbon shaped beam have the same phase of interference signal it is necessary that the gratings be aligned with respect to rotations about the beam axis to an angle of $\sim 10^{-5}$ rad. We have accomplished this by using a technique based on the optical polarizing properties of the gratings. The 0.2 μm-period grating lines act as wire grid polarizers for light. In principal, it would be possible to align two gratings by rotating them so as to maximize the amount of light transmitted through the pair. This is not practical because the transmitted intensity is proportional to the square of the relative angle between the gratings (for small angles), requiring intensity comparisons to a part in 10^{10}. However, if the polarization of the incident light is modulated about some center angle at frequency f, the amount of light transmitted with modulation frequency 2f, is linearly proportional to the angle between the grating and the center angle. We have used this technique[6] to align the gratings inside our machine to better than 10^{-4} rad, which will be sufficient for our purposes since we can afford to search through the final range of ~10 possible angles.

ATOM INTERFEROMETER EXPERIMENTS

We expect the atom interferometers will one day prove useful in the study of a number of problems in precision metrology, fundamental quantum mechanics, and atomic physics.

Metrology, especially General Relativity

In principal atom interferometers could be used in the manner of optical interferometers to measure fundamental quantities such as acceleration, length, and angular velocity. In practice, atom interferometers are very unlikely to be useful in the measurement of length or acceleration. This is because their advantages over optical interferometers are only due to the ratio of optical to atom de Broglie wavelength in the case of reflective interferometers (not likely to work for small λ_{db}), or on the ratio of optical wavelength to grating period in the case of diffractive interferometers. In either case these advantages will be outweighed by the superior fringe resolution and response time available from optical interferometers. In the area of basic metrology the promise of atom interferometers is in the sensing of inertial rotations, in this case both the low speed (compared to light) and the short wavelength of atoms are advantageous. The Sagnac effect sensitivity measured in radians of interferometer phase shift per unit of angular rotation frequency is $4\pi mA/h$ for a matter wave interferometer of area A, whereas it is $4\pi A/c\lambda$ for an optical interferometer operating at wavelength λ. For example, in order for rotation at one earth rate $\Omega_e \approx 10^{-5}$ sec^{-1} to cause a shift of one fringe in an interferometer using Xe atoms, it would need to have an enclosed area of 10^{-4} m^2, to achieve the same sensitivity in an interferometer using 0.5 μm light would require an area of 10^6 m^2. Of course, optical interferometers have the advantage that is is easy to fold the beam path so that the light makes many trips around the enclosed area effectively multiplying the sensitivity (and decreasing the frequency response) by the number of round trips. However, even if it were possible to build an optical ring cavity that had decay times equal to the millisecond transit times typical of atom interferometers, it would still be less sensitive by the wavelength ratio. It is worth noting that for interferometers using diffractive beam splitters at small incident angles (the simplest technology), the fact that $A \propto \lambda_{dB}$ means that the rotation sensitivity of the interferometer is inversely proportional to the atoms velocity; independent of the mass.

The obvious use for such precise rotation sensors is for tests of general relativity such as the search for the relativistic frame drag. The relativistic effects which might be observable with these techniques are as follows[7]: new limits on the preferred frame parameter in the PPN formalism ($\sim 10^{-8}\ \Omega_e$)[8], the velocity dependant frame drag ($\sim 10^{-9}\ \Omega_e$)[9], and the true Lense Thirring effect ($\sim 10^{-10}\ \Omega_e$)[9]. The second two of these effects are most easily measured by comparing an orbiting gyroscope to the position of the fixed stars as measured from a platform fixed to the gyro. The difficulty of measuring the frame drag can be appreciated when one considers that Everitt et al at Stanford[10] have been developing an experiment of this type (which employes a magnetically levitated spinning superconducting sphere as the gyro) for the last twenty years.

Fundamental Tests of Quantum Mechanics

Most of the experiments in fundamental quantum mechanics that have been performed using neutron interferometers could be improved by using atom interferometers. This is due both to the range of atomic properties potentially available and to the high brightness of atom sources as compared to neutron sources. We will consider two experiments that have not been performed with neutrons; an atom Hanbury Brown and Twiss experiment and a Berry's phase experiment with electric fields and integer spin particles.

Although not an interferometer in the same sense as described above, a conceptually simple application of atom beam splitters is the possibility of experimentally measuring the atom atom correlation functions in atom beams. The general picture of such experiments is shown is Figure 3, it is closely analogous the the Hanbury Brown and Twiss[11] experiment that measured second order correlations in photon counting. When performed using a 'classical' light source this experiment gives a coincidence rate at t=0 which is twice the rate at t∅± . This may be interpreted as photon bunching due to the Bose statistics of the electromagnetic field. There has been much recent interest in this phenomena which has centered around the production of anti-bunched states of the of the electromagnetic field in which the coincidence rate goes to zero at t=0. Correlation experiments with atoms would give access to quantum counting statistics in a fundamentally different regime: unlike photons, atoms are either bosons or fermions and is possible to define a positional wave function for atoms. One expect that given Δt (defined below) small enough the coincidence rate at t=0 will be zero for a beam of fermions and will be twice the rate at t∅± for a beam of Bosons from a thermal source.

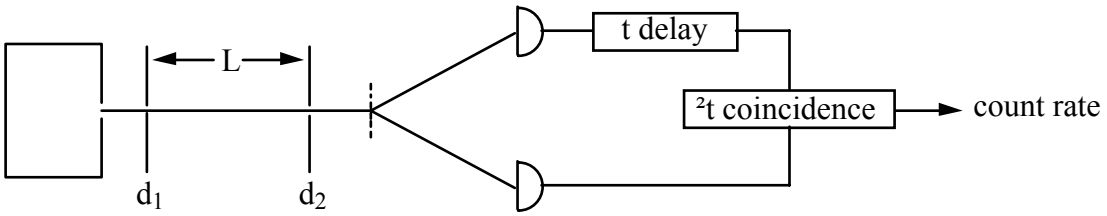

Figure 4. Schematic of an atom Hanbury Brown and Twiss experiment. d_1 and d_2 are the diameters of the collimation pin-holes.

The practical possibility of performing such an atom correlation experiment depends on the expected counting rate. We will calculate the coincidence counting rate as a function of the following beam parameters. At first we will assume that the experimental parameter Δt, the time window within which counts are registered as coincident can be chosen as small as is necessary.

Quantity	Symbol	Typical value
source brightness	B	10^{17}-10^{21} sr^{-1} sec^{-1} cm^{-2}
speed ratio	$s=v/\Delta v$	1-10^{-3}
mass	m	1-100 AMU
mean velocity	v	10^{2}-10^{5} cm sec^{-1}

We want the detectors to sample the same single transverse mode of the atom wave function. Therefore, the second aperture must fit within the diffraction pattern of the first i.e. $d_2=L\,\lambda_{dB}/d_1$, which implies that the flux through the second slit is given by (ignoring factors of $\pi/4$)

$$f=Bd_1^2\left(\frac{d_2^2}{L^2}\right)=B\lambda_{dB}^2$$

In addition to requiring that the detectors sample the same transverse mode we will also choose Δt to be equal to the atom coherence time τ so that the detectors sample a single longitudinal mode.

$$\tau=\frac{\lambda_{dB}}{v}s$$

If we assume that the counting rate is low, i.e. that the number of counts in Δt is small;

$$f\Delta t=\frac{\lambda_{dB}^3 S}{v}\ll 1$$

then the coincidence rate given by Poisson statistics is

$$f^2\Delta t=\frac{B^2Sh^5}{m^5v^6}\rightarrow\frac{10^{-12}B^2S}{m^5v^6}\ \text{(cgs with m in AMU)}$$

We believe that it will soon become experimentally feasible to measure the effects of atom-atom correlation in beams. An ideal system would be metastable He for which laser slowing techniques and fast detectors are readily available. For example; a laser slowed He^* source with a final velocity of 100 cm sec^{-1} and a brightness of 10^{13} sr^{-1} sec^{-1} cm^{-2} seems experimentally realizable, and would give a coincidence counting rate of 1 Hz. In this case the coherence time is $\tau=10^{-7}$ sec, and one can make Δt equal to (or even less than) τ by counting the He^* directly on electron multipliers.

Atom interferometers are an ideal system in which to investigate the predictions by M. V. Berry[12], Aharonov, and Anandan[13] regarding modifications to the adiabatic theorem. Despite the numerous recent tests of Berry's phase, atom interferometers allow a test of the theory which is novel in several respects. A Berry phase experiment involving the effect of electric fields on Na would be the first such experiment involving non-zero mass bosons and the first where the perturbing field appeared quadratically in the hamiltonian. It should also be possible to test the Aharonov-Anandan geometrical phase in the case of non-adiabatic change.

An atom interferometer measures any interaction which differentially affects the energies of particles traveling along separate paths through the interferometer. Thus, atom interferometers could be used to measure such quantities as the electric polarizability or magnetic susceptibility of atomic ground states, or to measure a basic null effect such as the charge neutrality of atoms. In order to determine what problems atom interferometers are most suitable for we must consider the factors which limit the precision of interferometric measurements. The relative precision with which a white fringe interferometer can be used to measure a differential energy shift ΔE is limited by the number of visible fringes, which is approximately given by the speed ratio (S) of the atom beam. The relative precision of energy measurement, $\Delta E/E$ is limited to $\Delta f/S$ where Δf is the fractional accuracy of fringe resolution. If the interferometer is shot noise limited $\Delta f \propto 1/\sqrt{n}$ where n is the total number of atoms counted to determine the phase of the interferometer fringe. These considerations suggest that atom interferometers may be most profitably employed as balance (null) meters i.e., when used to balance the effects of two different interactions applied to opposite sides of the interferometer. For example, an interferometer only slightly more advanced than our first device should be able to measure the ground state polarizability of sodium to $\sim 10^{-2}$ but, its ability to measure the ratio of polarizability to magnetic susceptibility would be limited only by the precision with which the strength of the individual fields could be controlled — perhaps two orders of magnitude better.

ATOM OPTICAL ELEMENTS

Most of the recent work in atom optics has involved the use of light pressure forces to manipulate the atom beams. Mirrors[14], lenses[15], and gratings for atoms have been demonstrated using the stimulated gradient forces on atoms in near-resonant optical radiation. We will not discuss these developments, instead we will review atom optical elements that do not involve light forces. we ignore light force atom optics both to contain the discussion and because of the obvious advantages of developing atom optical elements that are independent of laser technology. It is interesting to note that all the grating types (except the Kapitza-Dirac effect and the reflection/transmission grating) described above have been realized for neutrons and for X-rays. It is fruitful to look for alternative atomic optical elements based on the technology developed for x-ray optics. It should be possible to adopt grazing incidence x-ray mirrors, lenses, and diffraction gratings for use with atom beams. These techniques are based on the specular reflection of atoms from smooth surfaces, which may occur when the surface roughness is much less than the wavelength corresponding to the momentum of the atom perpendicular to the surface. For example, efficient specular reflection of reactive alkali atoms with thermal velocity at angles of up to 40 mrad has recently been reported by Haroche et al[16]. At least two reflective lenses for atoms have recently been demonstrated. Doak has made a cylindrical lens for a He beam by reflecting it off an Au coating on a bowed mica wafer at angles of about 30 degrees[17]. A most favorable system for demonstrating atomic reflection is the reflection of H off of films of He at cryogenic temperatures. Berkhout et al[18] have made a spherical mirror coated with liquid He that focuses an 18 mm diameter beam of H-atoms down to 0.5 mm. Further progress in reflective atom optics is hampered by the deficiency of theoretical or empirical knowledge of the necessary conditions for the reflection of atoms, especially slow atoms, from surfaces.

Another class of x-ray optical elements that could be adapted for use with atoms is based on transmission through micro-fabricated structures. Atom optical elements based on transmission have the advantage that they work for any atomic species independent of surface physics or laser technology. Since our demonstration of transmission diffraction gratings for atoms we have used similar methods[19] to produce 200 nm-period gratings as thin as 5 nm. These gratings can be tilted so as to increase their effective dispersive power, in addition they are a first step towards the reflection/transmission gratings described above. Fabrication methods similar to ours have been used to produce free standing zone-plates which should work as lenses for atom beams.

Slow sources

A key barrier to practical use of most of the atom optical devices discussed above is the poor brightness of existing slow atom sources. A number of radiation pressure atom slowers[20] have been demonstrated. They all work by arranging that an atom decelerating in the slower is continually exposed to radiation that is tuned slightly to the red of the atomic resonance and is directed opposite to the atomic velocity. This Doppler tuning condition may be met either by frequency chirping the laser or by Zeeman tuning the atom's resonance. In either case the atoms accumulate random transverse momentum due to the scattering of the incident photons, the rms transverse momentum is proportional to the square root of the number of photons needed to slow the atom times the total change in atom momentum.

The tools necessary to increase the brightness of slowed beams are available, but they have not as yet been assembled into a bright slow source. The simplest way to increase brightness is to apply transverse cooling in the form of 'red molasses' to the atoms emerging from the end of the slower. A more powerful general method for increasing brightness is to first apply transverse cooling followed by a lens (which alone, increases flux but not brightness), followed by a second region of transverse cooling at the focus of the lens. Another possibility would be to replace the cooler-lens-cooler combination with a single two dimensional spontaneous force optical trap. It is clear that there are no theoretical barriers to the development of laser slowed and intensified atom sources — the development of such sources is a worth while challenge for experimentalists in atom optics.The work on beam splitters was funded by the National Science Foundation (PHY86-05893) with help from the Joint Services Electronics Program (DAAL03-86-K-0002) which supports the M.I.T. Submicron Structures Laboratory. Work on the Atom interferometer is supported by O.N.R. (N0001489-J-1207) and A.R.O. (DAA L03-89-K-0082).

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