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Modelling the photophoretic force on a perforated membrane

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The photophoretic force exerted on an illuminated membrane vanishes when its size is much larger than the molecular mean-free-path. Perforations in the membrane can increase the force, but they make modelling the gas flow more difficult. In the present letter, a new model of gas interaction with a perforated surface is proposed. The direct simulation Monte Carlo method is used to calculate the flow-field and photophoretic force exerted on a perforated membrane, showing a significant increase of the force at high rarefaction parameters. This effect could be used to levitate membranes of few centimeters in size at mesospheric altitudes.

The phenomenon of photophoresis is caused by an illuminated object developing a temperature difference between its surfaces. This difference creates a movement of gas around the object, which imparts a force to the object [1]. The phenomenon has been deeply studied both experimentally and theoretically, see e.g. Refs.[2–9]. The main conclusion of these studies is that the photophoresis is determined by the pressure of surrounding gas and reaches its maximum value when the molecular mean-free-path is of the order of object size. Photophoresis causes small aerosols to rise at high altitudes, but the lofting force vanishes when the object size is much larger than the mean-free-path [10]. The question asked by Benford and Benford [11] is: could photophoresis be used to levitate something larger than an aerosol in the atmosphere?

The mean-free-path is small (about 1 mm) at mesospheric altitudes. However, a thin membrane with much greater size than the mean-free-path might levitate in the mesosphere if it contains many perforations smaller than the mean-free-path. If a temperature difference is established between both sides of the membrane, the perforations allow for thermal transpiration flow through the membrane, increasing the force towards the colder side as shown in Figure 1. Experimental observation of this phenomenon is growing. Duermann et al. [12] measured the photophoretic forces on porous circuit boards and tissue plates, showing that more pores lead to a larger force at a high pressure. Cortes et al. [13] fabricated 1 cm-wide hollow alumina sandwich structures with thin, rectangular channels that levitated under the equivalent intensity of as little as 5 suns at mesospheric pressures. Recently, we showed structures of similar size that approximate two freestanding, perforated membranes a small distance (125 μm) apart could levitate when exposed to the equivalent intensity of natural sunlight and the same pressures [14].

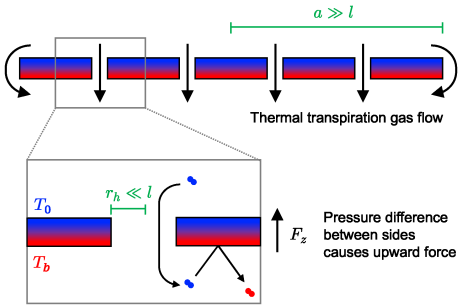

FIG. 1. Cross-section of a thin, perforated membrane with top surface temperature T_0, bottom surface temperature T_b, radius a and hole radius r_h. Inset: thermal transpiration gas flow from above the membrane, through the perforations, to the bottom of the membrane creates a pressure difference between the sides of the membrane.

Numerically modelling the photophoretic forces on these 2D plate-like structures is difficult because the force is relatively small and depends on many factors. Since the solution must be obtained for an arbitrary mean-free-path, the modelling should be based on rarefied gas dynamics [15]. The modelling becomes much more complicated when the plate is perforated, since a gas flow through thousands of holes spaced close to each other should be calculated.

The present letter proposes a simple method to model a gas flow through a perforated surface, avoiding hard calculations of gas flow through each single hole. Using the proposed method, the photophoretic force exerted on a circular perforated membrane is calculated considering that its radius is larger than the molecular mean-free-path. The results show that a perforation increases the lofting force by several times when the mean-free-path

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is around 100 times larger than the membrane radius. Since the method is not computationally costly, it is possible to perform the calculation for many degrees of the perforation and to point out its optimum value. In the future, the proposed approach can be use to calculate the lofting force for more sophisticated configurations.

Consider a membrane of radius a fixed in the plane xy with its center at $z = 0$. The membrane thickness is assumed to be much smaller than its diameter and the molecular mean-free-path. The membrane is surrounded by a gas kept at a temperature T_0 in an equilibrium state. The top membrane surface is kept at the equilibrium temperature T_0, while the bottom surface has a higher temperature $T_b > T_0$, which disturbers the gas. Under such conditions, the photophoretic force will push the membrane up, i.e., the lofting force will appear. This force is determined by the rarefaction parameter defined as

$$\delta = a/\ell, \quad \ell = \mu v_0/p_0, \quad v_0 = \sqrt{2R_g T_0/\mathscr{M}}, \qquad (1)$$

which is inversely proportional to the Knudsen number. Here, ℓ is the equivalent-free-path of gas molecules [15], μ is the dynamic viscosity of the gas, v_0 is the most probable speed of gaseous particles, R_g is the universal gas constant, $\mathscr{M}$ is the molar mass of the gas. The membrane is perforated so that it has many small holes distributed uniformly over its area. The hole radius r_h is always much smaller than the equivalent mean-free-path, i.e., $r_h \ll \ell$. Thus, the membrane is partially permeable and its porosity is defined as

$$\beta = \frac{N_h r_h^2}{a^2}, \qquad (2)$$

where N_h is the number of the holes. The lofting force F_z is expressed in terms of the dimensionless force $\mathscr{F}$ as

$$F_z = \mathscr{F}\frac{\pi a^2 p_0 (T_b - T_0)}{T_0}. \qquad (3)$$

It is expected that the dimensionless force $\mathscr{F}$ is weakly sensitive to many factors such as: species of the gas, gas temperature T_0, and the temperature difference $(T_b - T_0)$, but it strongly depends on the rarefaction parameter δ and porosity factor β. In accordance with our aim, we need to calculate the force $\mathscr{F}$ for large values of the rarefaction parameter δ and for several values of the porosity β.

The direct simulation Monte Carlo (DSMC) method [16] is applied to calculate the gas flow around the membrane and lofting force exerted on it. The intermolecular collisions are based on *ab initio* potential. The numerical scheme to model these collisions is the same for any type of gas flow. Its detailed description can be found in Refs.[17–20]. The main modification of the method is done in the gas-surface interaction modelling. To account for the membrane porosity, the following boundary conditions are adopted: a particle reaching the membrane surface is reflected from it with the probability $(1 - \beta)$, but the particle continues its motion with the probability β. The main novelty of the present work is the assumption that the membrane porosity is a microscopic property such that the shape and position of individual holes can be disregarded. This assumption is justified when the hole size is smaller than the equivalent free path, which is easily attained in practice.

When a particle is reflected, its new velocity is calculated according to the Cercignani-Lampis (CL) model [21] determined by two parameters α_t and α_n. The fist one α_t is the tangential momentum accommodation coefficient and the second one α_n is the normal energy accommodation coefficient. In the limit of $\alpha_t = \alpha_n = 1$, the scattering is diffuse, but both coefficients can be smaller than unity for light gases like helium [22–24]. As shown in Ref.[9], both α_t and α_n significantly affect the photophoretic force. In practice, the coefficients α_t and α_n depend on the gas species and surface state. They can be close to unity for heavy gases like krypton, but the coefficient α_n can be equal to 0.1 for helium interacting with non-treated surface [23], while its values drops to 0.01 for surfaces which are polished and treated by plasme [24]. To span more practical situations, both diffuse ($\alpha_t = \alpha_n = 1$) and non-diffuse ($\alpha_t = 0.9$, $\alpha_n = 0.1$) reflections are considered. The implementation of the CL model into the DSMC method is described in Ref.[15].

To mimic a boundless region around the membrane, it is assumed that all particles crossing the computational domain boundary are removed from further consideration, while the boundary itself emits particles with the equilibrium distribution function corresponding to the equilibrium pressure p_0 and temperature T_0.

The calculations were performed for helium for the following values of the rarefaction parameter $\delta = 5$, 10, 20, 50, 100 and for ten values of the porosity parameter β in the range $0 \le \beta \le 0.95$. The parameters of the numerical scheme are the same for all values of δ in the considered range $5 \le \delta \le 100$. The computational domain represents a cylinder of radius $R_D = 6a$ and height $H_D = 9a$. The cell size in both radial Δr and vertical Δz directions is equal to $\Delta r = \Delta z = 0.025a$. The time step Δt is equal to $0.01a/v_0$, and the number of model particles per cell is about 50. To provide a more uniform distribution of modelling particles over the radial direction, a weight depending on the radial coordinate is attributed to each particle. The number of samples to reach a reasonable statistical scattering depends on the rarefaction parameter δ. Finally, it was possible to reach the following numerical error including all its sources: 1% for $\delta \le 20$, 2% for $\delta = 50$, and 4% for $\delta = 100$.

The equilibrium temperature was assumed to be $T_0 = 300$ K, while the bottom temperature was $T_b = 400$ K. Since the quantity $\mathscr{F}$ drastically decreases by increasing rarefaction parameter δ, it is more convenient to plot the product $\delta\mathscr{F}$ versus the porosity β as is done in Fig-

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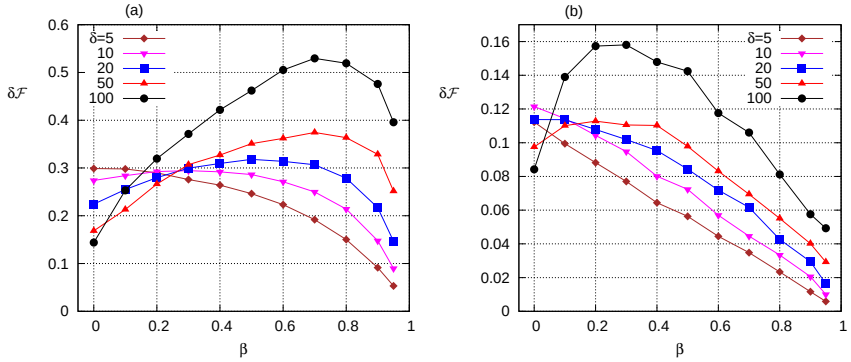

FIG. 2. Dimensionless force $\mathscr{F}$ vs. porosity β: (a) - diffuse and (b) - non-diffuse ($\alpha_t = 0.9$, $\alpha_n = 0.1$) reflections.

ure 2 (a) and (b), for diffuse and non-diffuse reflections, respectively. In the case of diffuse reflection, the perforations increase the lofting force more than three times at $\delta = 100$. The force $\mathscr{F}$ reaches its maximum value at about $\beta = 0.8$. When the rarefaction parameter decreases, the efficiency of the perforations decrease. For instance, the force increases due to the perforations just by 15% at $\delta = 20$. The perforations do not meaningfully increase the force at $\delta = 10$, while the force decreases by increasing β at $\delta = 5$. When the reflection is non-diffuse, the lofting force becomes quite smaller as predicted in the case of aerosols [9]. Introducing perforations to a surface reflecting non-diffusely increases the lofting force only by the factor 1.7, reached at $\beta = 0.3$. For a smaller rarefaction ($\delta \leq 50$), perforations do not significantly increase the force. In the range $\delta \leq 10$, perforations decrease the force. Such a behavior of the lofting force is consistent with the experimental data reported in Ref.[12].

To explain the increase of the lofting force due to perforations at a large value of the rarefaction, the temperature and speed distributions with streamlines are given in Figure 3 for the porosity $\beta = 0$ and 0.5. This picture shows why the photophoretic force exerted on an impermeable membrane is small. Indeed, the gas temperature below the membrane is close to that of the membrane bottom $T_b = 400$ K. Under such conditions, the momentum transfer from the gas to membrane is the same on both sides of the membrane. As a result, the pressures on both sides are the same and the photophoretic force is small. The gas temperature is different from the surface temperature only near the membrane edge. According to Figure 3(c), the gas is practically at rest near the membrane surface, but the bulk velocity of the gas near the membrane edge is high. Therefore, the lofting force exerting only on the membrane edge is relatively small. The flow-field for the perforated membrane is completely different, namely, the gas temperature near the membrane surface is quite different from that of the surface. This temperature difference leads to different pressures on the two sides of the membrane over its entire surface. Moreover, the gas flows through the membrane with a relatively large speed. Therefore, the photophoretic force exerted over the whole membrane surface is significant.

To have an idea of the order of magnitude of the lofting force F_z, its value calculated via Eq.(3) is plotted versus the pressure p given by Eq.(1) in Figure 4 (a) and (b) for diffusely and non-diffusely reflecting membrane of radius $a = 5$ cm, respectively. As expected, the force exerted on an impermeable membrane ($\beta = 0$) decreases by increasing the pressure, while a perforated membrane has the opposite trend for both diffuse and non-diffuse reflection. The force exerted on the perforated membrane with the diffuse reflection reaches the value 0.6 mN at $p = 50$ Pa that allows to levitate a membrane having the mass of 60 mg. For reference, the perforated membrane-like structures examined in Ref. [14] can be fabricated with area densities of around 1 g/m^2 and attain temperature differences $(T_b - T_0) \approx 100$ K at mesospheric pressures and solar illumination, suggesting atmospheric levitation is possible for cm-scale structures.

A new approach to model a rarefied gas flow through a perforated surface is proposed. The method avoids the difficult calculations of gas flow through individual holes in a membrane; instead the porosity is considered a microscopic characteristic of the membrane surface. Then, a gaseous particle approaching the surface can pass through it or can be reflected independently of the hit position. Using this assumption, the photophoretic force exerted on the membrane is calculated considering that the membrane radius is larger than the molecular mean-free-path. It was shown that perforations increase the photophoretic force several times under these conditions. Perforations in a membrane that diffusely reflects gas particles generate larger photophoretic forces than

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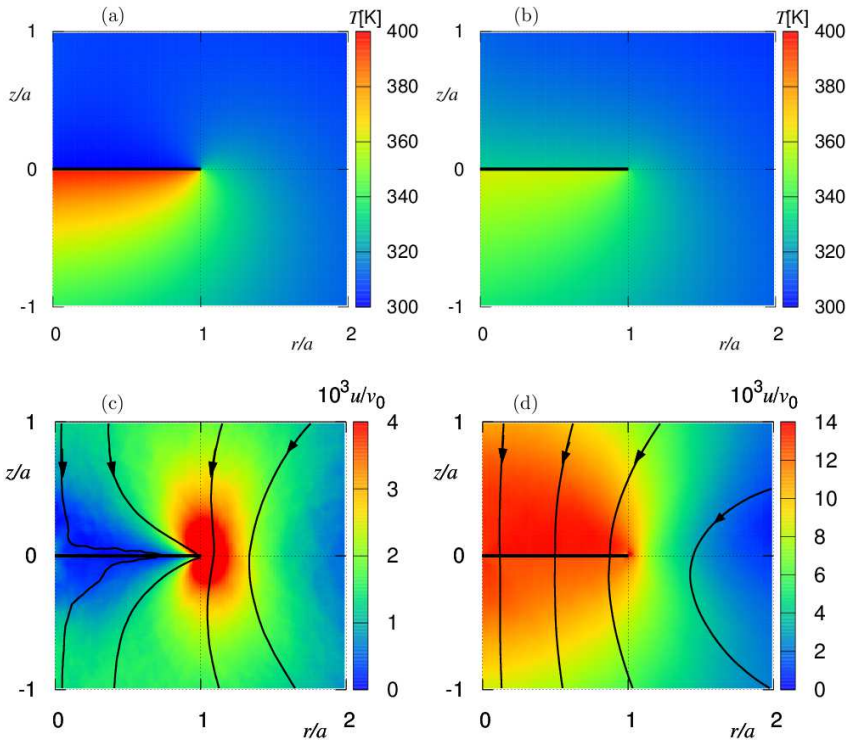

FIG. 3. Temperature T and speed distributions with streamlines at $\delta = 100$ and diffuse reflection: (a) and (b) - temperature; (c) and (d) - speed; (a) and (c) - $\beta = 0$; (b) and (d) - $\beta = 0.5$

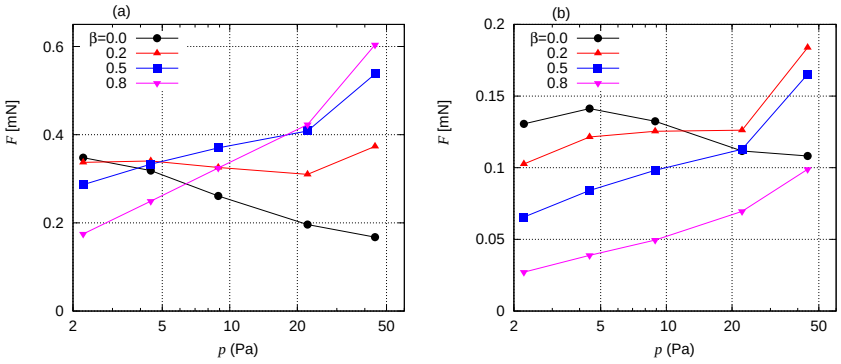

FIG. 4. Lofting force F versus pressure p for a membrane of $a = 5$ cm: (a) - diffuse and (b) - non-diffuse reflections.

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on a membrane that reflects non-diffusely. It was shown that the photophoretic phenomenon can be used to levitate thin membranes few centimeters in size. In a forthcoming full-length paper, additional factors that affect the lofting force such as the chemical composition of gas, the top and bottom temperature of the membrane, and smaller gas rarefaction are analyzed. Moreover, the heat flux from the membrane to the surrounding gas will be also reported. The proposed method can be used to find the optimum characteristics of a membrane with maximum force. It can also be applied to more sophisticated and realistic levitating objects, such as a double-layer membrane with both freestanding holes and cylindrical channels connecting the layers.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

AUTHOR CONTRIBUTIONS

Felix Sharipov: Conceptualization (equal); Software (lead); Formal analysis (equal); Writing – original draft (lead); Writing – review & editing (equal).

Benjamin Schafer: Conceptualization (equal); Investigation (lead); Formal analysis (equal); Writing – original draft (supporting); Writing – review & editing (equal).

David Keith: Conceptualization (equal); Project administration (lead); Formal analysis (equal); Writing – original draft (supporting); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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